What is the best way to slice a convex polytope?

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I will discuss two old classical problems in computational geometry:

1. Given a d-dimensional convex polytope P, what is the best slice of P by a hyperplane? Here best can mean many possible things, e.g., a slice with the largest volume? Or a slice with the largest number of vertices? etc. This touches on classical work by Bourgain, Ball, Koldobsky, Milman, and many other mathematicians.

2. As we slice P with hyperplanes we create many combinatorially different $(d-1)$-slices, which are also polytopes of course. E.g., for a 3-dimensional regular cube there are 4 combinatorial types of slices (triangles, quadrilaterals, pentagons, hexagons). How many different ones are there for a polytope P? How can we count them all? Can we give lower/upper bounds on their number? What are extremal cases?

I will explain a powerful new algorithmic framework that answers these problems (and others) in polynomial time when $\dim(P)$ is fixed. Moreover, we show the problems have hard complexity otherwise. This is joint work with Marie-Charlotte Brandenburg (MPI/KTH) and Chiara Meroni (Harvard/ETH).