Forbidding subgraphs in the hypercube

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One of the central problems in graph theory is finding, for a given graph H, the extremal function $ex(n, H)$, that is the largest number of edges in an n-vertex graph that contains no isomorphic copy of H as a subgraph. While determining the asymptotic behaviour of $ex(n, H)$ remains a challenge in general, we know exactly what graphs have positive Turán density, i.e., for what graphs H is $ex(n, H)$ a positive proportion of the total number of edges on n vertices. An analogous function $ex(Q_n, H)$, the largest number of edges in a subgraph of the n-dimensional hypercube Q_n that contains no isomorphic copy of H, is much less understood. In particular, we even do not have any characterisation for graphs H that have a positive hypercube Turán density. In this talk I will report on some recent progress on $ex(Q_n, H)$ and show connections between this function and other problems in extremal combinatorics.